# Entanglement dynamics and Bell Violations of two atoms in Tavis-Cummings model with phase decoherence *

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## Abstract

Considering the dipole-dipole coupling intensity between two atoms and the field in the Fock state, the entanglement dynamics between two atoms that are initially entangled in the system of two two-level atoms coupled to a single mode cavity in the presence of phase decoherence has been investigated. The two-atom entanglement appears with periodicity without considering phase decoherence, however, the phase decoherence causes the decay of entanglement between two atoms, with the increasing of the phase decoherence coefficient, the entanglement will quickly become a constant value, which is affected by the two-atom initial state, Meanwhile the two-atom quantum state will forever stay in the maximal entangled state when the initial state is proper even in the presence of phase decoherence. On the other hand, the Bell violation and the entanglement does not satisfy the monotonous relation, a large Bell violation implies the presence of a large amount of entanglement under certain conditions, while a large Bell violation corresponding to a little amount of entanglement in certain situations. However, the violation of Bell-CHSH inequality can reach the maximal value if two atoms are in the maximal entangled state, or vice versa.

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## I. INTRODUCTION

Quantum entanglement is one of the most striking features of quantum mechanics, and plays an important role in quantum information processing, such as quantum teleportation[1], quantum dense coding[2], quantum cryptography[3] and quantum computation[4]. Therefore quantum entanglement has been viewed as an essential resource for quantum information process, and a great deal of effort has been devoted to study and characterize the entanglement. Cavity quantum electrodynamics (QED) techniques has been recognized as a promising candidate for the physical realization of quantum information processing. Quantum entanglement based cavity QED was generated by sending two atoms being present simultaneously in the cavity [5] or the two atoms interacting consecutively with the cavity [6]. However, the above preparation processes are considered in closed system and the influences of environment are neglected. Time evolution of isolated quantum systems is followed by the Schrodinger equation. But a quantum system unavoidably interacts with the environment. The decoherence effect of this interaction will lead to the degradation of quantum coherence and entanglement. The entangled state will loss purity and become mixed. Entanglement dynamics behavior of a quantum system coupled to its environment can reflect the details of the decoherence effect[7,8]. On the other hand, entanglement can exhibit the nature of a nonlocal correlation between quantum systems. Bell's theorem[9] provides a effective way to test quantum nonlocality[10], quantum nonlocality will be exhibited if Bell-type inequality is violated for a given quantum state. Namely, a violation of any Bell-type inequality gives a quantitative confirmation that a state behaves quantum nonlocality.

In the original papers, researchers investigated the entanglement in the JCM[11], a damped JCM[12] and two-atom Tavis-Cummings model[13]. Recently Hein etal.[14] investigate entanglement properties of multipartite states under the influence of decoherence. Reference [7] shows that quantum mechanical entanglement can prevail in noisy open quantum systems at high temperature and far from thermodynamical equilibrium, despite the deteriorating effect of decoherence. Reference [8]considers the interaction of a single two-level atom with one of two coupled microwave cavities and shows analytically that the atom-cavity entanglement increases with cavity leakage.We investigate the entanglement time evolution of two entangled two-level atoms that interact resonantly with a single-mode field in the Fock state[15]. In Ref.[16], the author investigated two two-level atoms coupled to a single mode optical cavity with the phase decoherence and showed the rich dynamical features of entanglement arising between atoms and cavity or between

two atoms, however the two-atom dipole-dipole coupling intensity is neglected, the two atoms are initially in a separate state and the cavity field is initially prepared in the vacuum state. In order to study explicitly the entanglement dynamics of the two-atom system, therefore, in this paper we investigate the entanglement dynamics between two atoms that are initially in entangled state in Tavis-Cummings model introducing dipole-dipole coupling intensity and the field in the Fock state with phase decoherence, to our knowledge, which has not been reported so far. In addition quantum nonlocality has been widely studied for the two-atom entanglement system using Bell-CHSH inequality. Our studies show that the entanglement between two atoms and Bell-CHSH inequality decay with phase decoherence and disappear in a constant, which is affected by two-atom initial state and dipole-dipole coupling intensity. Meanwhile many new interesting phenomena are exhibited, e.g., the two-atom quantum state will forever stay in the maximal entangled state when the initial state is proper even in the presence of phase decoherence. These interesting phenomena result from two-atom initial state and dipole-dipole coupling intensity. The phase decoherence can be used to play a constructive role and generate the controllable stable entanglement by adjusting two-atom initial state and dipole-dipole coupling intensity.

This paper is organized as follows. We introduce the model and calculate the reduced density matrices of two two-level atoms in the next section. In Sec. 3, Entanglement dynamics of two atoms with phase decoherence have been studied. Sec. 4 gives the relations between entanglement and Bell violations, and Sec. 5 is the conclusions.

## II. THE MODEL AND REDUCED DENSITY MATRICES OF TWO TWO-LEVEL ATOMS

Consider two two-level atoms interacting resonantly with a single-mode cavity field initially prepared in the Fock state. In the rotating-wave approximation the Hamiltonian of the atom-field system reads

$$H = \omega_0 \sum_{j=1}^{2} S_j^z + \omega_a a^\dagger a + \sum_{j=1}^{2} g(a^\dagger S_j^- + a S_j^+) + \sum_{i,j=1;i\neq j}^{2} \Omega S_i^+ S_j^- \tag{1}$$

where $a$ ($a^\dagger$) denotes the annihilation (creation) operator of the resonant single-mode field, $\omega_0$, $\omega_a$ are atomic transition frequency, cavity frequency, respectively, $g$ is the coupling constant between atoms and cavity, $S_j^+ = |e\rangle_j\langle g|$, $S_j^- = |g\rangle_j\langle e|$, $S_j^z = \frac{1}{2}(|e\rangle_j\langle e| - |g\rangle_j\langle g|)$ are atomic operators, and $\Omega$ is atomic dipole-dipole coupling constant. In this paper, we investigate the entanglement between two atoms by only considering the phase decoherence. In this situation, the master equation

governing the time evolution of the system under the Markovian approximation is given by[17]

$$\frac{d\rho}{dt} = -i[H,\rho] - \frac{\gamma}{2}[H,[H,\rho]] \tag{2}$$

where$\gamma$ is the phase decoherence coefficient. The equation with the similar form has been proposed to describe the intrinsic decoherence [18]. The formal solution of the master equation (2) can be expressed as follows [19]:

$$\rho(t) = \sum_{k=0}^{\infty} \frac{(\gamma t)^k}{k!} M^k(t)\rho(0)M^{\dagger k}(t) \tag{3}$$

where $\rho(0)$ is the density operator of the initial atom-field system and $M^k(t)$ is defined by

$$M^k(t) = H^k exp(-iHt)exp(-\frac{\gamma t}{2}H^2) \tag{4}$$

We assume $\omega_0 = \omega_a$, the cavity field is prepared initially in the Fock state $|n\rangle$, atom A and atom B are prepared in the entangled state $\cos\theta|eg\rangle + \sin\theta|ge\rangle$, then the initial density operation for the whole atom-field system is

$$\rho(0) = (\cos\theta|eg\rangle + \sin\theta|ge\rangle)(\cos\theta\langle eg| + \sin\theta\langle ge|) \otimes |n\rangle\langle n| \tag{5}$$

In the subspace of $K = a^\dagger a + \frac{1}{2}(S_1^z + S_2^z) \equiv n$, the eigenvectors and eigenvalues of Hamiltonian (1) can be written as[20]

$$\begin{aligned}
|E_0\rangle &= -\sqrt{\frac{1+n}{1+2n}}|n-1\rangle|ee\rangle + \sqrt{\frac{n}{1+2n}}|n+1\rangle|gg\rangle, E_0 = n\omega \\
|E_1\rangle &= \frac{1}{\sqrt{2}}(|n\rangle|ge\rangle - |n\rangle|eg\rangle), E_1 = n\omega - \Omega \\
|E_2\rangle &= \frac{1}{2}\sqrt{\frac{\Delta-\Omega}{\Delta}}(\frac{4\sqrt{n}g}{\Delta-\Omega}|n-1\rangle|ee\rangle - |n\rangle|ge\rangle - |n\rangle|eg\rangle + \frac{4\sqrt{n+1}g}{\Delta-\Omega}|n+1\rangle|gg\rangle), \\
E_2 &= \frac{1}{2}(2n\omega + \Omega - \Delta) \\
|E_3\rangle &= \frac{1}{2}\sqrt{\frac{\Delta+\Omega}{\Delta}}(\frac{4\sqrt{n}g}{\Delta+\Omega}|n-1\rangle|ee\rangle + |n\rangle|ge\rangle + |n\rangle|eg\rangle + \frac{4\sqrt{n+1}g}{\Delta+\Omega}|n+1\rangle|gg\rangle), \\
E_3 &= \frac{1}{2}(2n\omega + \Omega + \Delta)
\end{aligned} \tag{6}$$

Where $\Delta = \sqrt{8(1+2n)g^2 + \Omega^2}$

Substituting $\rho(0)$ into the Eq.(3), the exact time-dependent density operation can be expressed as

$$\begin{aligned}
\rho(t) =& C_1|E_1\rangle\langle E_1| + C_2|E_2\rangle\langle E_2| + C_3|E_3\rangle\langle E_3| + C_4|E_1\rangle\langle E_2| + \\
& C_5|E_2\rangle\langle E_1| + C_6|E_1\rangle\langle E_3| + C_7|E_3\rangle\langle E_1| + C_8|E_2\rangle\langle E_3| + C_9|E_3\rangle\langle E_2|
\end{aligned} \tag{7}$$

where

$$C_1 = \frac{1}{2}(1-\sin 2\theta), C_2 = \frac{1}{4}(1+\sin 2\theta)\frac{\Delta-\Omega}{\Delta}, C_3 = \frac{1}{4}(1+\sin 2\theta)\frac{\Delta+\Omega}{\Delta}$$
$$C_4 = \frac{1}{2\sqrt{2}}\cos 2\theta\sqrt{\frac{\Delta-\Omega}{\Delta}}exp(-\frac{(E_2-E_1)^2}{2}\gamma t)exp(i(E_2-E_1)t)$$
$$C_5 = \frac{1}{2\sqrt{2}}\cos 2\theta\sqrt{\frac{\Delta-\Omega}{\Delta}}exp(-\frac{(E_2-E_1)^2}{2}\gamma t)exp(-i(E_2-E_1)t)$$
$$C_6 = -\frac{1}{2\sqrt{2}}\cos 2\theta\sqrt{\frac{\Delta+\Omega}{\Delta}}exp(-\frac{(E_3-E_1)^2}{2}\gamma t)exp(i(E_3-E_1)t)$$
$$C_7 = -\frac{1}{2\sqrt{2}}\cos 2\theta\sqrt{\frac{\Delta+\Omega}{\Delta}}exp(-\frac{(E_3-E_1)^2}{2}\gamma t)exp(-i(E_3-E_1)t)$$
$$C_8 = \frac{1}{\sqrt{2}}(1+\sin 2\theta)\frac{g\sqrt{1+2n}}{\Delta}exp(-\frac{(E_3-E_2)^2}{2}\gamma t)exp(i(E_3-E_2)t)$$
$$C_9 = \frac{1}{\sqrt{2}}(1+\sin 2\theta)\frac{g\sqrt{1+2n}}{\Delta}exp(-\frac{(E_3-E_2)^2}{2}\gamma t)exp(-i(E_3-E_2)t)$$

The reduced density matrices of the subsystem composed of two two-level atoms is

$$\rho_{AB}(t) = a_1|gg\rangle\langle gg| + a_2|ge\rangle\langle ge| + a_3|ge\rangle\langle eg| + a_4|eg\rangle\langle ge| + a_5|eg\rangle\langle eg| + a_6|ee\rangle\langle ee| \quad (8)$$

Where

$$a_1 = (1+\sin 2\theta)\frac{2(n+1)g^2}{\Delta^2}(1-exp(-\frac{(E_3-E_2)^2}{2}\gamma t)\cos(E_3-E_2)t) \quad (9)$$

$$\begin{aligned} a_2 =& \frac{1}{2} + (1+\sin 2\theta)\frac{(1+2n)g^2}{\Delta^2}(-1+exp(-\frac{(E_3-E_2)^2}{2}\gamma t)\cos(E_3-E_2)t) \\ &- \frac{1}{4}\cos 2\theta\frac{\Delta-\Omega}{\Delta}exp(-\frac{(E_2-E_1)^2}{2}\gamma t)\cos(E_2-E_1)t \\ &- \frac{1}{4}\cos 2\theta\frac{\Delta+\Omega}{\Delta}exp(-\frac{(E_3-E_1)^2}{2}\gamma t)\cos(E_3-E_1)t \end{aligned} \quad (10)$$

$$\begin{aligned} a_3 = a_4^* =& \frac{\sin 2\theta}{2} + (1+\sin 2\theta)\frac{(1+2n)g^2}{\Delta^2}(-1+exp(-\frac{(E_3-E_2)^2}{2}\gamma t)\cos(E_3-E_2)t) \\ &- \frac{i}{4}\cos 2\theta\frac{\Delta-\Omega}{\Delta}exp(-\frac{(E_2-E_1)^2}{2}\gamma t)\sin(E_2-E_1)t \\ &- \frac{i}{4}\cos 2\theta\frac{\Delta+\Omega}{\Delta}exp(-\frac{(E_3-E_1)^2}{2}\gamma t)\sin(E_3-E_1)t \end{aligned} \quad (11)$$

$$\begin{aligned} a_5 =& \frac{1}{2} + (1+\sin 2\theta)\frac{(1+2n)g^2}{\Delta^2}(-1+exp(-\frac{(E_3-E_2)^2}{2}\gamma t)\cos(E_3-E_2)t) \\ &+ \frac{1}{4}\cos 2\theta\frac{\Delta-\Omega}{\Delta}exp(-\frac{(E_2-E_1)^2}{2}\gamma t)\cos(E_2-E_1)t \\ &+ \frac{1}{4}\cos 2\theta\frac{\Delta+\Omega}{\Delta}exp(-\frac{(E_3-E_1)^2}{2}\gamma t)\cos(E_3-E_1)t \end{aligned} \quad (12)$$

$$a_6 = (1 + \sin 2\theta)\frac{2ng^2}{\Delta^2}(1 - exp(-\frac{(E_3 - E_2)^2}{2}\gamma t)\cos(E_3 - E_2)t) \quad (13)$$

## III. ENTANGLEMENT DYNAMICS OF TWO ATOMS WITH PHASE DECOHERENCE

In order to discuss the entanglement dynamics in the above system, we adopt the negative eigenvalues of the partial transposition to quantify the degree of entanglement. The idea of this measure of the entanglement is the Peres-Horodecki criterion for the separability of bipartite systems [21]. The state is separable if the partial transposition is a positive operator, however, if one of the eigenvalues of the partial transposition is negative then the state is entangled. For a two-qubit system described by the density operator, the negativity can be defined by:[22]

$$E_{AB} = -2\sum_i \mu_i \quad (14)$$

where $\mu_i$ are the negative eigenvalues of the partial transposition of $\rho_{AB}^{\Gamma}$. When $E_{AB} = 0$, the two qubits are separable and $E_{AB} = 1$ indicates maximal entanglement between them.

We can make a partial transposition for atom B and work out the eigenvalues of the partial transposition $\rho_{AB}^{\Gamma}$. The four eigenvalues are $a_2$, $a_5$, $\frac{1}{2}(a_1 + a_6 + \sqrt{a_1^2 - 2a_1a_6 + a_6^2 + 4a_3a_4})$, $\frac{1}{2}(a_1 + a_6 - \sqrt{a_1^2 - 2a_1a_6 + a_6^2 + 4a_3a_4})$. Substitute them into Eq.(14), the explicit expression of $E_{AB}$ characterizing the entanglement of two atoms can be found to be

$$\begin{aligned} E_{AB} &= |a_2| + |a_5| + |\frac{1}{2}(a_1 + a_6 + \sqrt{a_1^2 - 2a_1a_6 + a_6^2 + 4a_3a_4})| + \\ &|\frac{1}{2}(a_1 + a_6 - \sqrt{a_1^2 - 2a_1a_6 + a_6^2 + 4a_3a_4})| - (a_2 + a_5 + \frac{1}{2}(a_1 + a_6 + \sqrt{a_1^2 - 2a_1a_6 + a_6^2 + 4a_3a_4}) \\ &+ \frac{1}{2}(a_1 + a_6 - \sqrt{a_1^2 - 2a_1a_6 + a_6^2 + 4a_3a_4})) \end{aligned} \quad (15)$$

In the following, we analyze the numerical results for the time evolution of the two-atomic entanglement.

We firstly consider the case of $\gamma = 0$, i.e., the absence of phase decoherence. The time evolution behaviors of the entanglement are showed in Fig.1-Fig.3(assuming g=1 in all the figures in this paper) with different initial state and dipole-dipole coupling intensity for $g = 1, n = 0$.

Figure 1 depicts the time evolution of the entanglement when the pair of atoms are initially prepared in the different states. It is observed that the entanglement evolves periodically in the absence of phase decoherence. We consider three cases of the initial state, i.e., the disentangled of the two atoms ((a)and(c), solid line ), not maximal entangled state ((a), (b), (c) and(d), dashed

line) and maximal entangled state ((b)and(d), solid line). In the first case, we can observe that the two atoms that are initially separate can generate entanglement by the atom-field interaction and atom-atom interaction. At certain time the entanglement evolves to its zero and the two two-level atoms are disentangled, while at the large time scale the two atoms are entangled. In a period, the degree of the entanglement increases gradually to a larger value(about 0.5), then decreases to a smaller value(about 0.2), then again increases and finally decreases to zero. In the second case, the two atoms own the same entanglement at $t = 0$, but have different phase angles. It is the phase angle that leads to considerable different time evolution of the entanglement. One case is that the degree of the two-atom entanglement is no more than that of the initial entanglement, as is shown in Fig.1((a)and (b), dashed line), the other case is that the degree of the two-atom entanglement is more than that of the initial entanglement all the time during the interaction, the peak of the entanglement increases, as is shown in Fig.1((c)and (d), dashed line), which means the larger entangled state can be prepared by choosing the initial phase angle. The third case is that the two atoms are initially in the maximal entangled state. In Fig.1((c), solid line), the time evolution is similar to the above case, however, from Fig.1((d), solid line), we can find the two-atom quantum state will forever stay in the maximum entangled state when the initial state is proper, this corresponding to the fact that the two atoms do not show any dynamic evolution and remain the initial state.

Figure 2 displays the time evolution of the entanglement for two values of no and weak dipole-dipole interaction. Fig.2((a), solid line) corresponding to the case of being no dipole-dipole interaction, the peak of the maximum entanglement becomes small comparing with the case of that in Fig.1((a),solid line, $\Omega = 1$), Since there is no dipole-dipole interaction between the two atoms, it is very clear that this entanglement is induced purely by atom-field interaction. This is consistent with Ref. [11]. The dipole-dipole interaction plays a constructive role in the entanglement formation between two atoms. From these figures, we can see that the degree of the entanglement is not necessarily increases with the increase of dipole-dipole interaction. In Fig.2(c), the degree of the entanglement can reach the maximum value 1 and the range of the oscillation becomes larger comparing with the situation in Fig.1(a), while the value of the dipole-dipole interaction in Fig.2(c) is less than that in Fig.1(a). It is interesting to find that the two atoms can generate maximal entangled state even they are separate initially by adjusting the dipole-dipole interaction.

In Fig.3, we consider the situation of strong dipole-dipole interaction. With the increase of dipole-dipole interaction, the period of the oscillation becomes short. The time evolution character is similar to the case of the weak dipole-dipole interaction for the separate initial state. However,

for the entangled initial state, that is not the case. An interesting result is the entanglement between the two atoms increases to a larger value than the initial entanglement in Fig.3((a)and (c), dashed line), while the entanglement decreases in in Fig.1((a), dashed line). In the strong coupling case, i.e., $\Omega \gg g$, from Eq.(9-13), we can see that dipole-dipole interaction $\Omega$ plays a key role in the quantum entanglement between the atom. Atom-atom interaction reduces the atom-field interaction. That is to say, strong dipole-dipole interaction is helpful for the entanglement production.

Let us now turn to discuss the condition of existing phase decoherence($\gamma \neq 0$). The phase decoherence causes the decay of the entanglement between two atoms, which is shown in Figs.4(a)and 4(b). With the increase of phase decoherence coefficient, the initial entanglement oscillates with time and will gradually become a constant value, which depends on the initial state of the two atoms. That is to say, the phase decoherence in the atom-field interaction suppresses the entanglement, but the phase decoherence can not fully destroy the entanglement between two atoms. From Figs.4(a)and 4(b), we can also see that the pairwise entanglement between two atoms can achieve a very large value even in the presence of phase decoherence, which is similar to the case without phase decoherence. For the proper initial state, their entanglement can be preserved during the time evolution as its initial value with phase decoherence. The above time evolution character arises due to in the time evolution the additional term in Eq.(2) leads to the appearance of the decay factor, which are responsible for the destruction of the entanglement. In order to discuss how the entanglement changes with the dipole-dipole interaction, in Figs.4(c)and 4(d) we give the plot of the entanglement for $\Omega = 0.5$ and $\Omega = 5$ in the present of $\gamma = 0.1$. The result is that more stronger the dipole-dipole interaction is, more faster the entanglement does oscillate. As for the situation of strong dipole-dipole interaction, the entanglement decreases rapidly, then approaches to a stable value, which is different from the case in the absent of phase decoherence. what affects the stable value? From Eq.(15), it is easy to verify that $E_{AB}$ in the case of $\gamma \neq 0$ for given long time,

$$E_{AB} = \frac{-2(1+2n)g^2(\sin\theta + \cos\theta)^2 + \sqrt{4g^4(\sin\theta+\cos\theta)^4 + (1+2n)^2(-2g^2+6(g^2+\Omega^2)\sin 2\theta)^2}}{8(1+2n)g^2+\Omega^2} \tag{16}$$

which means that the entanglement of stationary state depends on the initial state, the dipole-dipole interaction and the field in the Fock state. One may question whether there exists a situation in which two atoms can forever achieve maximal entanglement in the present of phase decoherence. Fig.4((a)and(b), dash dot line) give the answer. What is the reason why the two atoms can

stay the maximal entanglement in the present of phase decoherence ? From Eq.(9-13), we can see $a_1 = a_6 = 0$, $a_2 = a_5 = \frac{1}{2}$, $a_3 = a_4 = -\frac{1}{2}$ if the angles satisfy the following relation $\theta = \frac{(4k-1)\pi}{4}, k = 1, 2, \cdots$. The two atoms are in the maximally entangled state $\frac{1}{\sqrt{2}}(eg\rangle - |ge\rangle)$, so the entanglement has nothing with the phase decoherence coefficient, the two-atom initial state, the dipole-dipole coupling intensity between two atoms and the field in the Fock state.

At the end of this section, we discuss to achieve entanglement between the two atoms if the initial atoms are prepared in different states and the cavity field is in the Fock state. In Fig.5, we plot the entanglement as the function of time t for different values of phase decoherence rate $\gamma$ and dipole-dipole coupling intensity $\Omega$ if the field in the $|1\rangle$. Two cases are shown in Fig.5(a) for different dipole-dipole coupling intensity if $\gamma = 0$, i.e the entanglement between two atoms of being no dipole-dipole interaction falls off while $E_{AB}$ increases having dipole-dipole interaction as n increases for the initial separate two-atom state. The influence is completely different compared to that for the $n = 0$ case. For the initial entangled two-atom state, the notable difference here is that the peak of the entanglement becomes larger than that in Fig.1, while at some time, the two atoms stay in the separate state. The photon number n helps to increase the peak value of entanglement. Figs.5(c) and 5(d) corresponding to the case of phase decoherence $\gamma = 0.1$. An interesting comparison can be made with the case of the field in the vacuum state. The entanglement decays sharply as n increases and the stationary state entanglement is affected by the Fock state, so we can get two-atom entanglement mediated by the Fock state cavity field.

From the above analysis, it is clear to note that the phase decoherence coefficient, the two-atom initial state, the dipole-dipole coupling intensity between two atoms and the field in the Fock state have notable influence on the entanglement of two atoms.

## IV. BELL VIOLATIONS AND THE RELATIONS BETWEEN ENTANGLEMENT AND BELL VIOLATIONS

The quantum nonlocal property can be characterized by the maximal violation of Bell's inequality. Jeong etal.[23] have defined the maximal violation of Bell's inequality as mearurement of the degree of quantum nonlocality. Here we discuss the CHSH inequality. The CHSH operator is defined by[24]

$$\vec{B} = (\vec{a} \cdot \vec{\sigma}) \otimes (\vec{b} \cdot \vec{\sigma}) + (\vec{a} \cdot \vec{\sigma}) \otimes (\vec{b'} \cdot \vec{\sigma}) + (\vec{a'} \cdot \vec{\sigma}) \otimes (\vec{b} \cdot \vec{\sigma}) + (\vec{a'} \cdot \vec{\sigma}) \otimes (\vec{b'} \cdot \vec{\sigma}) \qquad (17)$$

where $\vec{a}$, $\vec{a'}$, $\vec{b}$, $\vec{b'}$ are unit vectors. The hidden variable theories impose the Bell-CHSH inequality $| < \vec{B} > | \leq 2$ where $< \vec{B} >$ is the mean value of the bell operation for a given quantum state. However, in the quantum theory it is found that $| < \vec{B} > | \leq 2\sqrt{2}$, which implies the Bell-CHSH inequality is violated. The maximal amount of Bell's violation of a state $\rho$ is given by [25]

$$< B >= 2\sqrt{\lambda + \lambda'} \tag{18}$$

Where $\lambda$, $\lambda'$ are the two largest eigenvalues of $T_\rho^\dagger T_\rho$, the elements of matrix $T_\rho$ are $(T_\rho)_{nm} = Tr(\rho\sigma_n \otimes \sigma_m)$, here $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, and $\sigma_3 = \sigma_z$ denote the usual Pauli matrices. For the density operator in Eq. characterizing the time evolution of two atoms, $\lambda + \lambda'$ can be written as follows:

$$\lambda + \lambda' = 4a_3a_4 + \max[4a_3a_4, (a_1 + a_6 - a_2 - a_5)^2] \tag{19}$$

it is easy to draw the violation of Bells inequality for two atoms.

$$< B >= 2\sqrt{4a_3a_4 + \max[4a_3a_4, (a_1 + a_6 - a_2 - a_5)^2]} \tag{20}$$

Similarly, Figs.6-8 display the numerical results of the analytical expression of maximal violation of Bell's inequality for the field in the vacuum state. In Fig.6, we plot the time evolution of the maximal violation of Bell's inequality for $\Omega = 1$ and $\Omega = 0.5$ when the two atoms are prepared in different states. For the sepatate initial state, our calculations show that two atoms cannot violate the CHSH inequality in this case, which is seen in Fig.6((a)dashed line). If we appropriately choose the value of the dipole-dipole interaction $\Omega$, From Fig.6(c)(dashed line), an interesting result is that two atoms can violate the CHSH inequality in certain time. Even the two atoms have the same entanglement and the phase angle, it is the dipole-dipole interaction that makes the CHSH inequality of the two atoms evolve in different ways. The violation of the CHSH inequality increases firstly in Fig.6(b)(solid line), while the violation the CHSH inequality decreases firstly in Fig.6(d)(dashed line). In addition the violation of Bell-CHSH inequality can stay in the maximal value when the entanglement angle satisfies $\theta = 3\pi/4$. Fig.7 corresponding to the time evolution of Bell-CHSH inequality in the present of phase decoherence. Fig.8 depicts the time evolution of Bell-CHSH inequality against the strong dipole-dipole interaction with the phase decoherence and without the phase decoherence. The result is expected as it is shown in Figs.8(a) and 8(b) that the strong dipole-dipole interaction maximize the violation of the CHSH inequality, in this case the larger violation of Bell-CHSH inequality can be achieved. Similar to the influence of phase decoherence on the entanglement, the violation of Bell-CHSH inequality is

very fragile against the phase decoherence and finally disappears in the different stationary state with different initial state and dipole-dipole coupling intensity.

In the following, we are devoted to settling the relationship between entanglement, measured in terms of the negativity, and the Bell violations in the system [1]. And although the quantitative relations have never been investigated in detail, it is quite often suggested that a large Bell violation implies the presence of a large amount of entanglement and vice versa. Recently, Verstraete et al. investigated the relations between the violation of the CHSH inequality and the concurrence for systems of two qubits[26]. For the pure states and some Belldiagonal states, the maximal value of B for given concurrence C is $2\sqrt{1+C^2}$. If the concurrence $C \geq \sqrt{2}/2$, the minimal value of B is $2\sqrt{2}C$, furthermore, the entangled two-qubits state may not violate any CHSH inequality with the concurrence C$\leq \sqrt{2}/2$, except their Belldiagonal normal form does violate the CHSH inequalities. Comparing Fig.1((a) solid line) with Fig.5((a) solid line), we can find that though two atoms get entangled in the time evolution, two atoms cannot violate the CHSH inequality in this case. Fig.5 shows two atoms can violate the CHSH inequality in the case that the entanglement is larger than a certain value. Under certain condition, the more Bell violation, the larger amount of entanglement. However, the violation of Bells inequality is not a sufficient condition for the entanglement, that is to say, a large Bell violation is not necessarily with a large amount of entanglement, which can be seen in Fig.1((c) dashed line), Fig.2((d) solid line), Fig.6((b)solid line, (d)dashed line). In Fig.3((a) solid line) and Fig.8((a) solid line). The dipole-dipole interaction decreases the degree of violation while increases the amount of entanglement. One interesting point is that the entanglement degree is initially very little, while the violation of Bells inequality can be generated, according to Re.[23], we can know the Bell diagonal normal form in system (1) does violate the CHSH inequalities. Our calculations also show that the condition of the maximal violation is that the entanglement degree is maximal. In a word, the Bell violation and entanglement does not satisfy the monotonous relation. This is consist with Re.[13]. So this phenomenon is still valid for the form of Bell's inequality and the entanglement measurement in this paper.

## V. CONCLUSION

In summary, we have studied quantum entanglement and quantum nonlocality of two atoms in Tavis-Cummings model with phase decoherence. It is shown that the phase decoherence causes the decay of entanglement between two atoms. With the increasing of the phase decoherence coefficient, the entanglement will quickly become a constant value, which is affected by the two-atom initial state, the dipole-dipole coupling intensity and the field in the Fock state. Therefore, the amount of the entanglement can be increased by adjusting the two-atom initial state, the dipole-dipole coupling intensity and the field in the Fock state. The violation of Bell-CHSH inequality is very fragile against the phase decoherence and finally disappears in the different stationary state in the absence of phase decoherence. In addition, the relationship between the entanglement and the nonlocality of two atoms is investigated, under certain conditions either a larger violation or a less violation can be generated with the increasing of entanglement. We hope that the results obtained in this paper would find their applications in quantum information processing and the test of quantum nonlocality.


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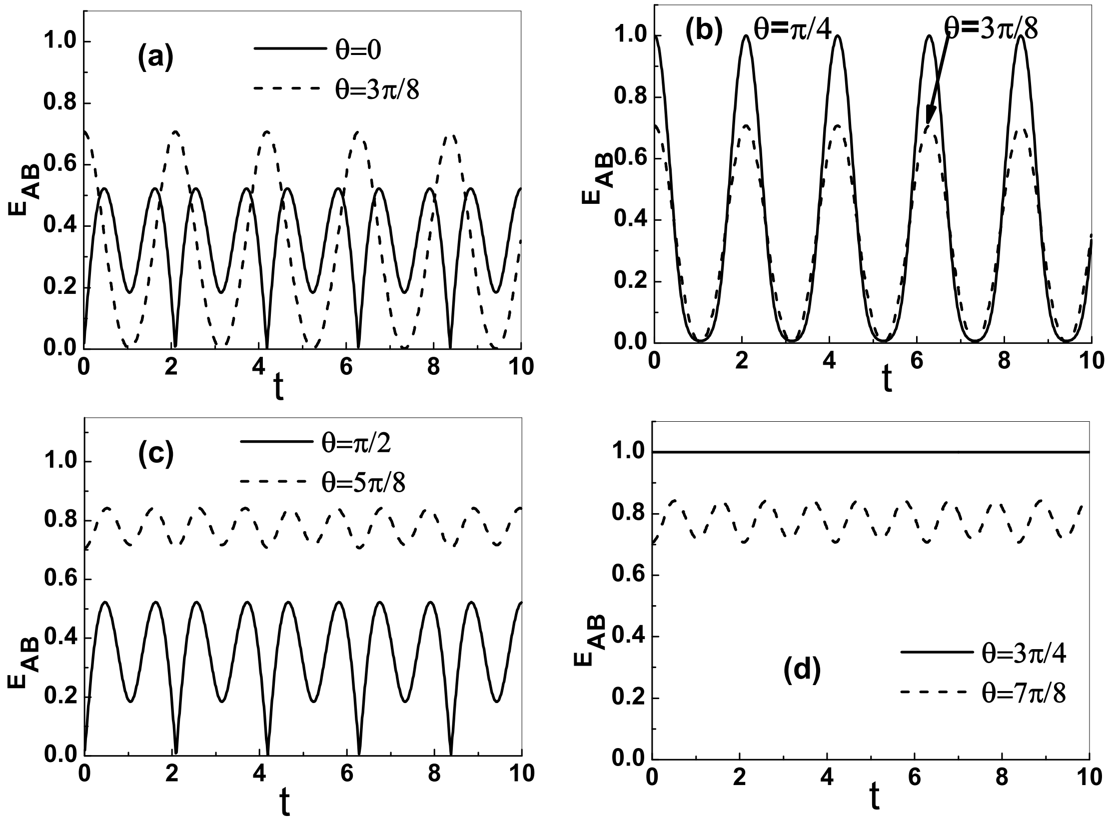


FIG. 1: The entanglement between the two atoms ($E_{AB}$) is plotted as a function of time t with $g = 1, \Omega = 1, \gamma = 0, n = 0$ when the two-atomic state is initially prepared in the different state

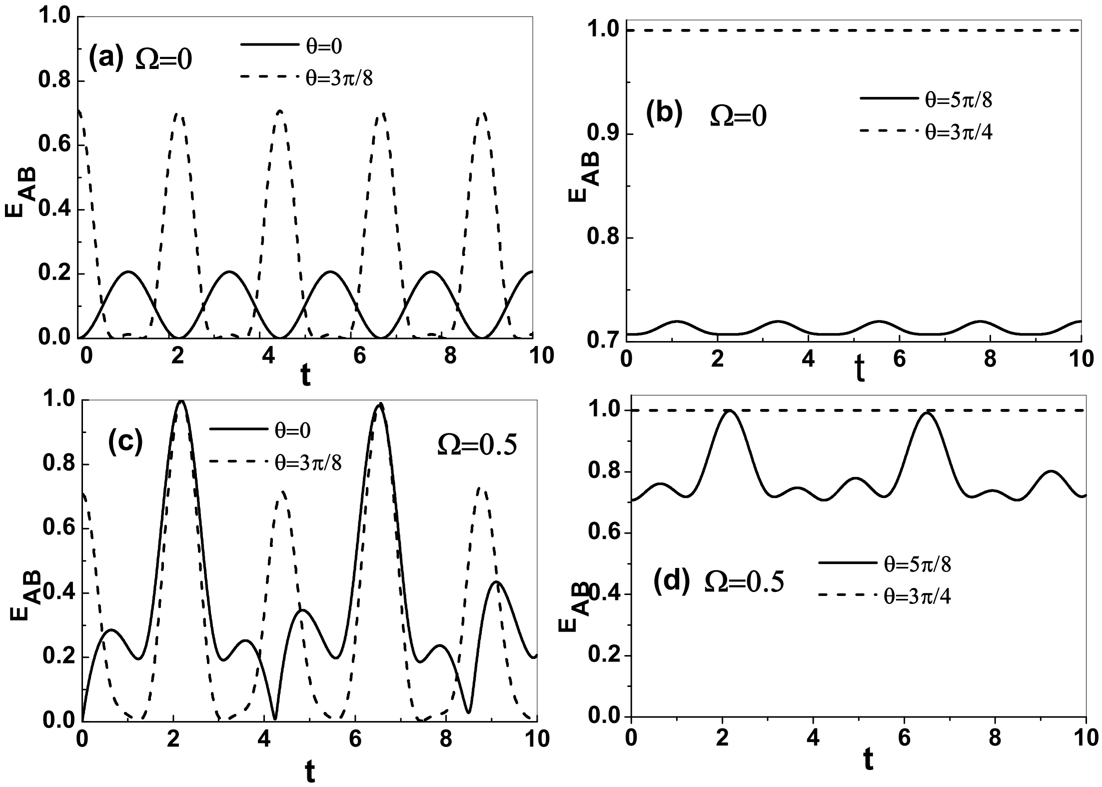


FIG. 2: The entanglement between the two atoms ($E_{AB}$) is plotted as a function of time t with $g = 1, \gamma = 0, n = 0$.

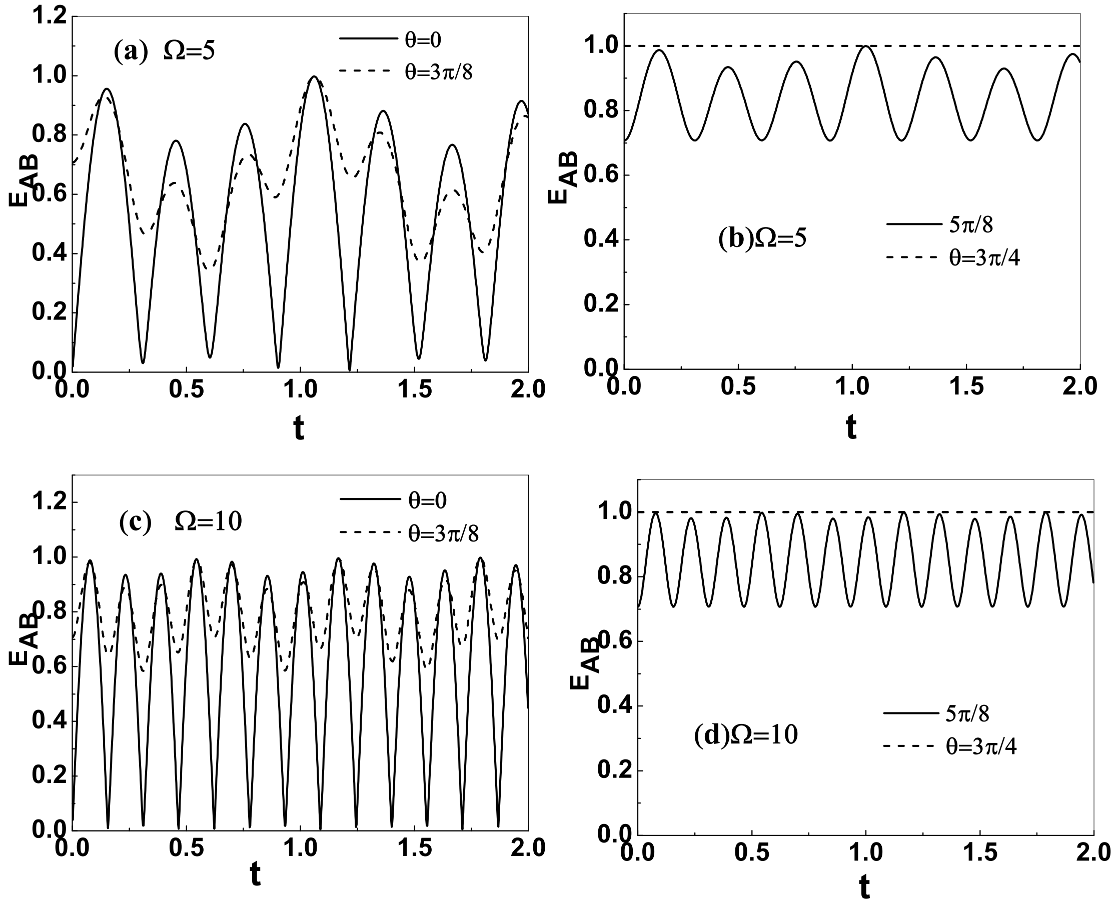


FIG. 3: The entanglement between the two atoms ($E_{AB}$) is plotted as a function of time t with $g = 1, \gamma = 0, n = 0$.

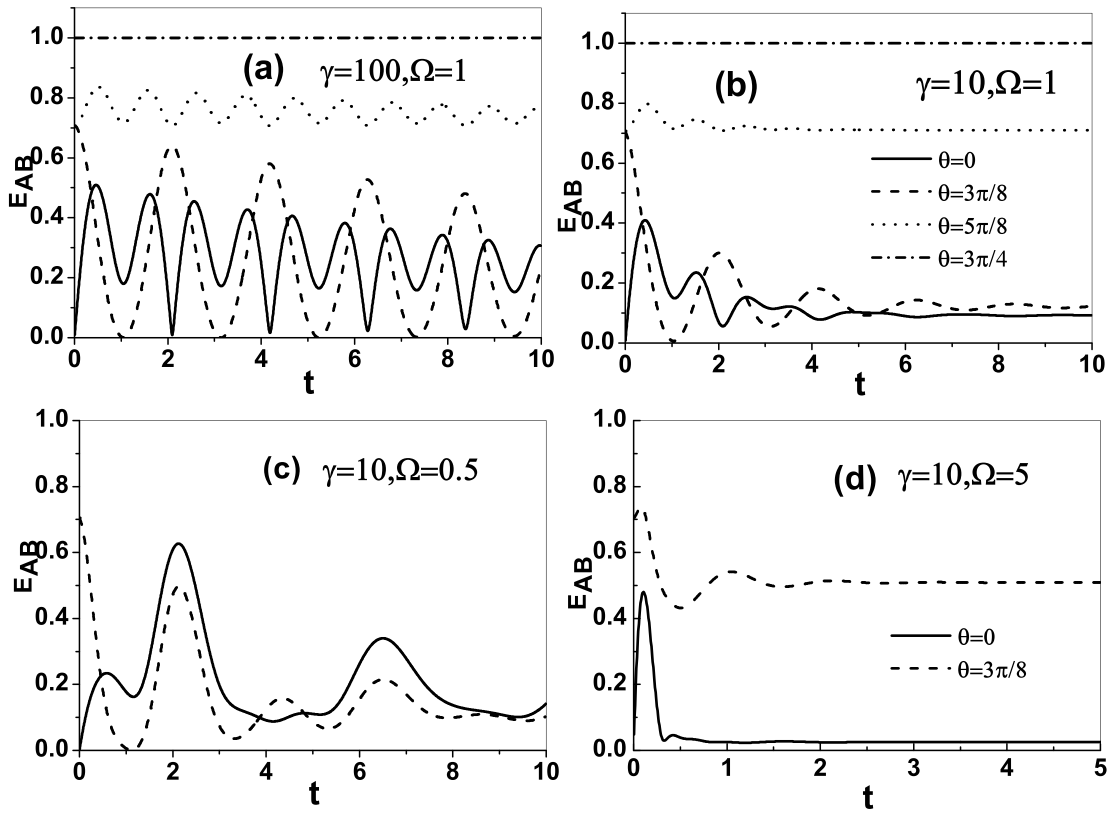


FIG. 4: The entanglement between the two atoms ($E_{AB}$) is plotted as a function of time t with $g = 1, n = 0$ in the present of phase decoherence.

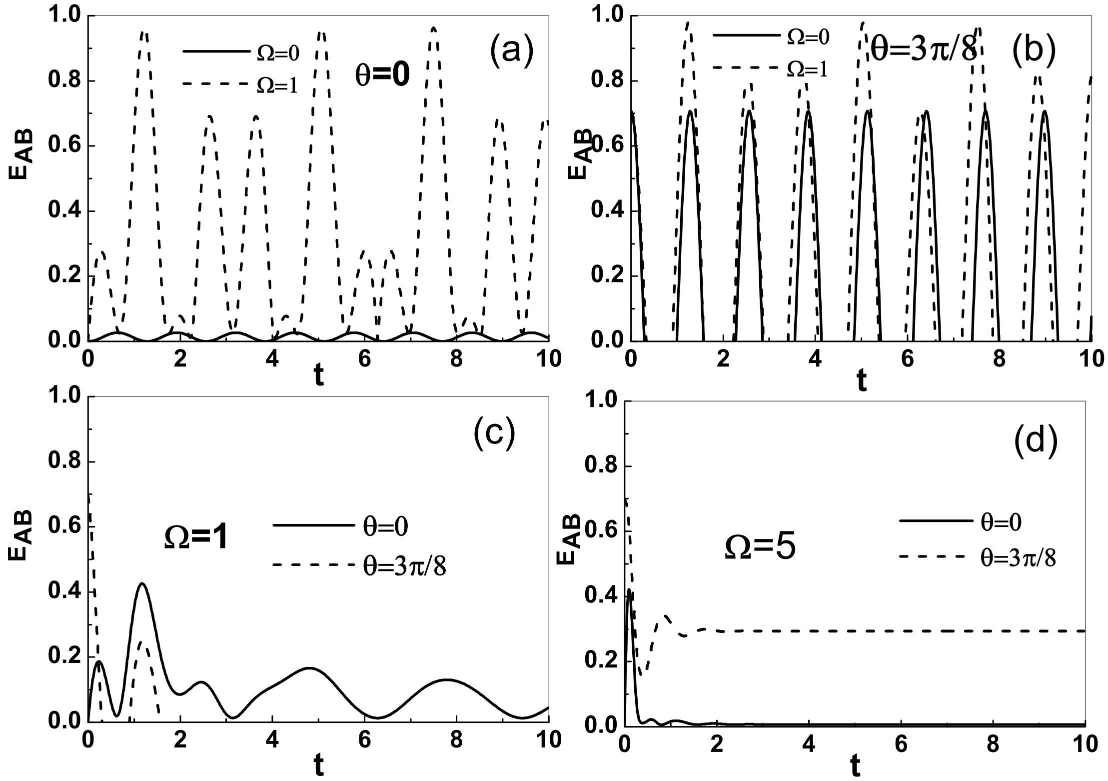


FIG. 5: TThe entanglement between the two atoms ($E_{AB}$) is plotted as a function of time t with $g = 1, n = 1$, (a) and (b)$\gamma = 0$, while (c) and (d)$\gamma = 0.1$

Fig.6 The time evolution of maximal violation of Bell-CHSH inequality for $g = 1, \gamma = 0, n = 0$.

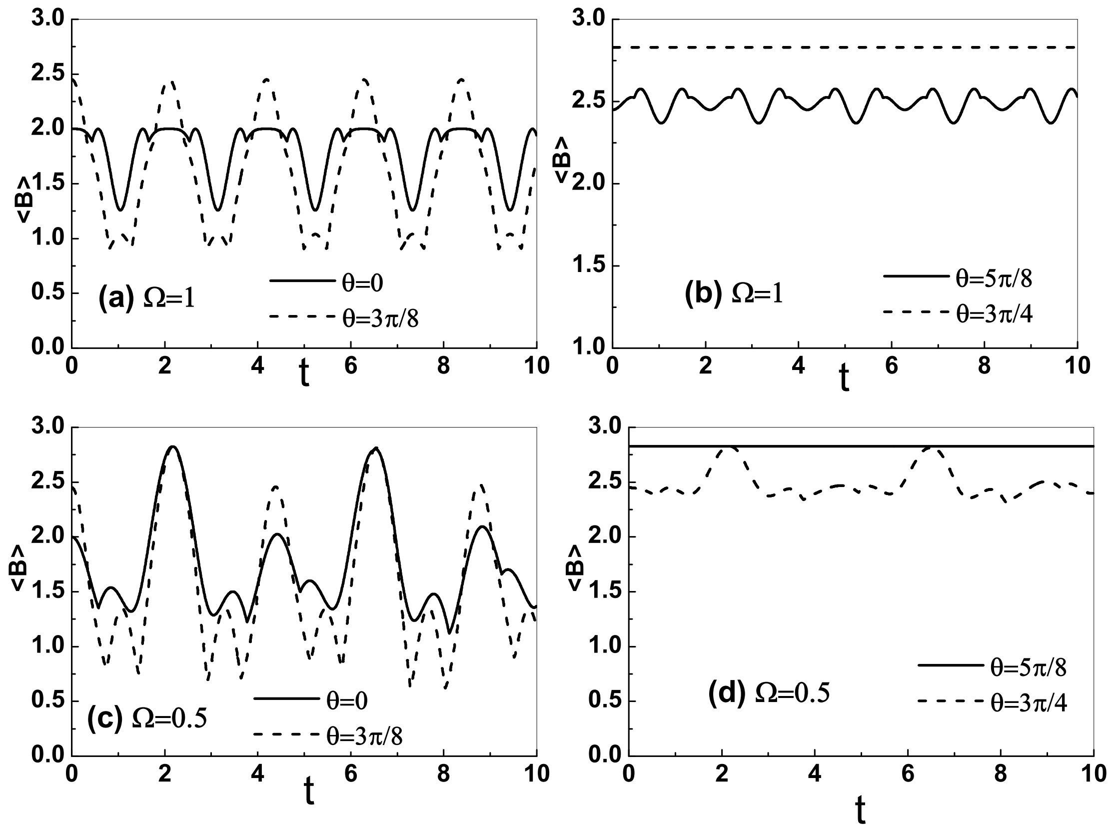


FIG. 6: The time evolution of maximal violation of Bell-CHSH inequality for $g = 1, \gamma = 0, n = 0$.

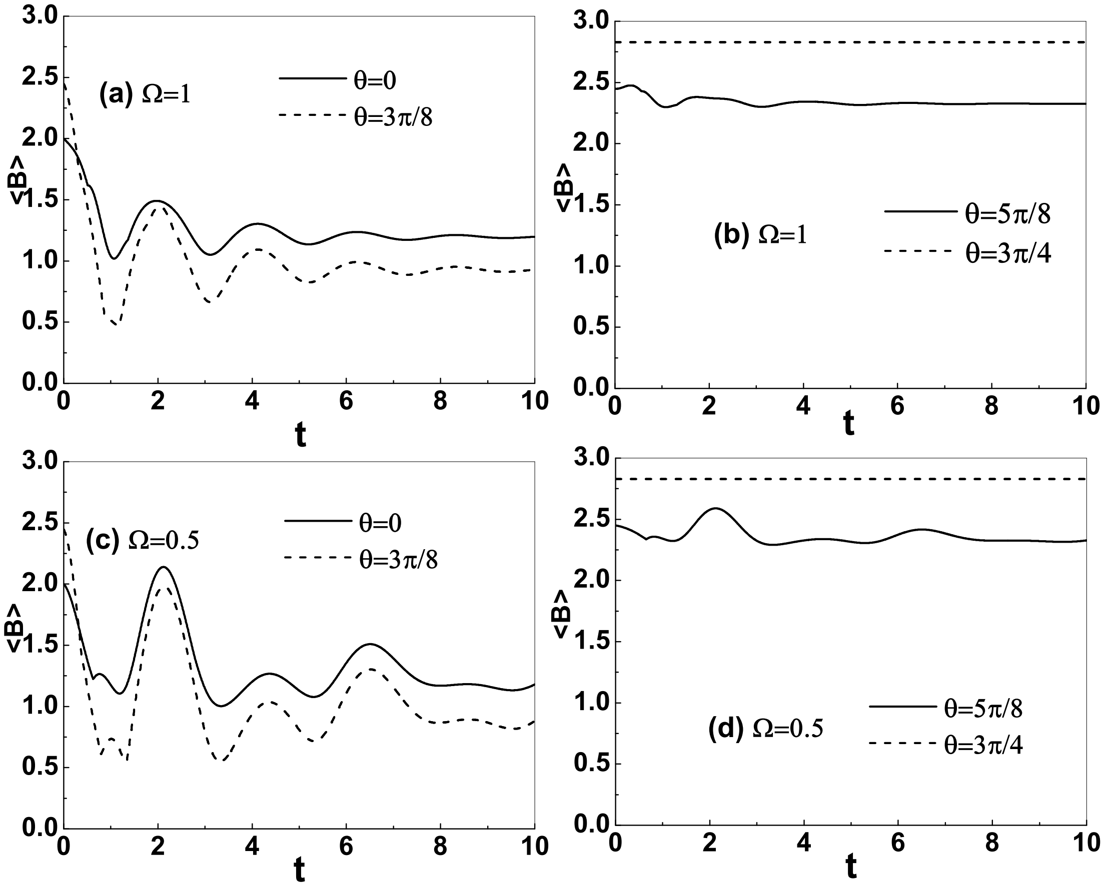


FIG. 7: The time evolution of maximal violation of Bell-CHSH inequality for $g = 1, \gamma = 0.1, n = 0$.

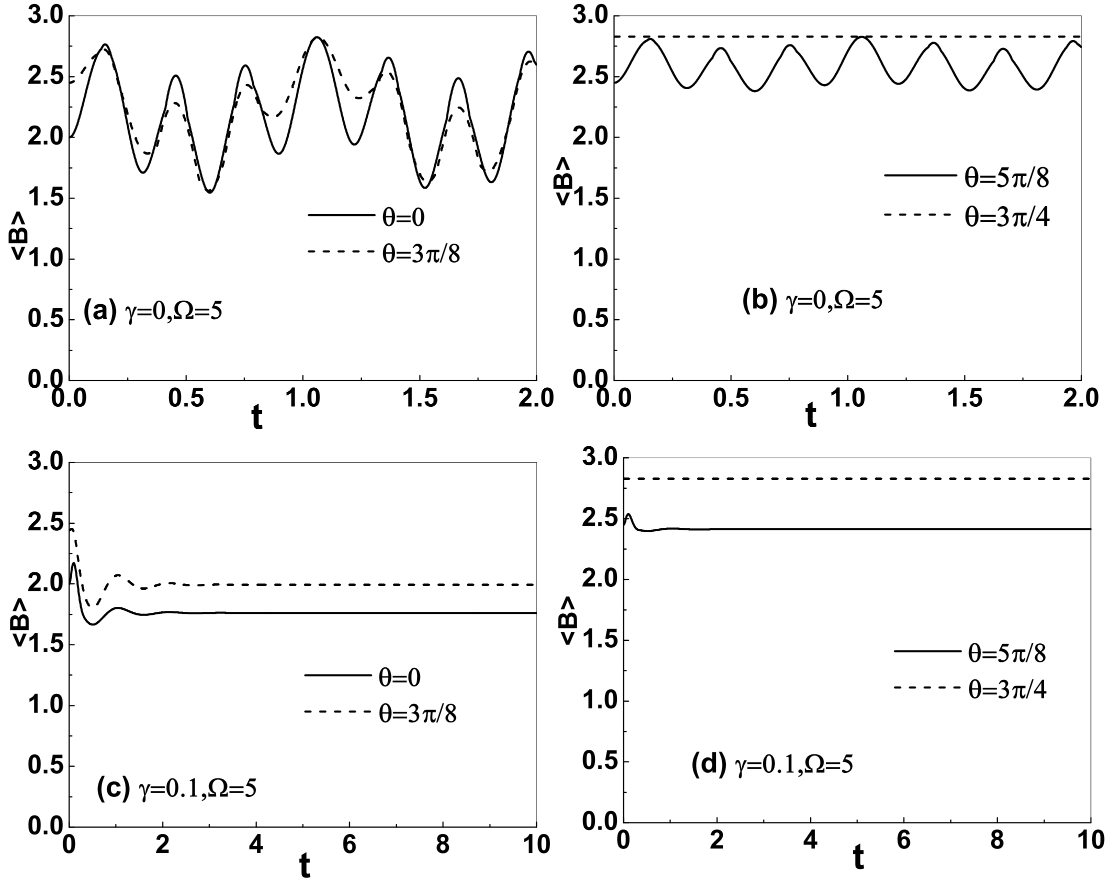


FIG. 8: The time evolution of maximal violation of Bell-CHSH inequality for $g = 1, \Omega = 5, n = 0$.